

Hořava Gravity after GW170817

A. Emir Gümrükçüoğlu,¹ Mehdi Saravani,² and Thomas P. Sotiriou^{2,3}

¹*Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX, UK*

²*School of Mathematical Sciences, University of Nottingham, University Park, Nottingham, NG7 2RD, UK*

³*School of Physics and Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, UK*

The recent detection of gravitational wave GW170817 has placed a severe bound on the deviation of the speed of gravitational waves from the speed of light. We explore the consequences of this detection for Hořava gravity.

I. INTRODUCTION

Hořava gravity [1] has been proposed as an ultraviolet (UV) complete theory for the gravitational interaction. The improved behaviour at high energies is due to the presence of higher-order terms in spatial derivatives and this requires violations of local Lorentz invariance. The action of Hořava gravity is [1, 2]

$$S = \frac{M_p^2}{2} \int N dt \sqrt{h} d^3x \left(K_{ij} K^{ij} - \lambda K^2 + \xi^{(3)} R + \eta a_i a^i + \frac{1}{M_\star^2} \mathcal{L}_4 + \frac{1}{M_\star^4} \mathcal{L}_6 \right). \quad (1)$$

N , N^i , and g_{ij} are the lapse function, the shift and the induced metric on a given spacetime foliation by spacelike hypersurfaces, while K_{ij} is their extrinsic curvature. \mathcal{L}_4 and \mathcal{L}_6 contain terms that are fourth-order and sixth-order in spatial derivatives, respectively. Hence, they contribute fourth and sixth powers of momenta in the dispersion relations. These contributions are suppressed at momenta below some scale M_\star , where the theory becomes effectively an infrared (IR) modification of General Relativity (GR), but dominate in the UV and are expected to render interactions renormalizable. The action is not invariant under the full group of diffeomorphisms, but only under diffeomorphisms that respect the foliation. This underscores that the foliation is preferred.

The first line of eq. (1) can be thought of as the IR limit of Hořava gravity. Elevating the space-time symmetry to full diffeomorphisms by introducing a Stückelberg field ϕ , one can write the IR action as [3]

$$S_{IR} = \frac{M_\star^2}{2} \int \sqrt{-g} d^4x \left[{}^{(4)}R + \alpha u^\mu u^\nu \nabla_\mu u_\alpha \nabla_\nu u^\alpha - \beta \nabla_\mu u_\nu \nabla^\nu u^\mu - \gamma (\nabla_\mu u^\mu)^2 \right], \quad (2)$$

where we defined $u_\mu \equiv -\nabla_\mu \phi / \sqrt{-\nabla^\nu \phi \nabla_\nu \phi}$. This action coincides with the first line of action (1) after the partial gauge fixing $\phi = t$ and with the following correspondence of parameters through

$$\lambda = \frac{1+\gamma}{1-\beta}, \quad \eta = \frac{\alpha}{2(1-\beta)}, \quad \xi = \frac{1}{1-\beta}, \quad M_p^2 = (1-\beta) M_\star^2. \quad (3)$$

Moreover, the covariant formulation is equivalent to a restricted version of the Einstein-Æther theory [4], in which the æther is forced to be hypersurface-orthogonal at the level of the action (*i.e.* before the variation) [5]. The correspondence of parameters is given in Appendix A.

Hořava gravity propagates two tensor and one scalar polarization. All three polarizations satisfy higher-order dispersion relations, as mentioned earlier. Their speeds in the infrared limit are

$$c_T^2 = \frac{1}{1-\beta}, \quad c_S^2 = \frac{(2-\alpha)(\gamma+\beta)}{\alpha(1-\beta)(2+3\gamma+\beta)}. \quad (4)$$

The recent detection of a binary neutron star merger with coincident gamma ray emission has introduced remarkably strong constraints on c_T [6]. The purpose of this brief note is to discuss the implications of this constraint for Hořava gravity and to clarify how this constraint can be effectively combined with existing ones. It has recently been pointed out in Ref. [7] that the speed of the scalar polarisation is almost unconstrained and our results highlight that this feature persists even after GW170817.

It is worth pointing out that our focus is on infrared viability. Hence, the higher order terms contained in \mathcal{L}_4 and \mathcal{L}_6 in action (1) will not be relevant and we will not attempt to give them explicitly. We will, however, discuss constraints on M_\star and mention how theoretical considerations related to the UV properties of the theory can restrict M_\star . We will also not consider any version of Hořava gravity where additional restrictions of the action are considered in order to reduce the numbers of independent couplings, *e.g.* [8–12]. Even though some of these restricted models have been shown to have interesting properties — the so called ‘projectable’ theory [8], for instance, has been shown to be renormalizable beyond power-counting [13]¹ — they also tend to suffer from infrared viability issues [16–22].

II. DIRECT CONSTRAINTS AND BOUNDS

In this section we list all of the available constraints in terms of the parameters (α, β, γ) of action (2).

¹ Projectable theory in 2+1 dimensions [14] has actually been shown to be asymptotically free [15].

1. Unitarity. The kinetic term for the scalar mode should have the same sign as the kinetic term of the tensor modes in order for the Hamiltonian to be bounded for linearised perturbations around flat space. This yields [2]:

$$\frac{2 + 3\gamma + \beta}{\gamma + \beta} > 0. \quad (5)$$

2. Perturbative stability. The coefficients of the gradient terms should have the right sign for stability [2]:

$$0 < \alpha < 2, \quad \beta < 1. \quad (6)$$

This condition, along with the previous one ensures that c_T^2 and c_S^2 are always positive.

3. BBN. Cosmology provides further constraints. On a cosmological background, the effective gravitational constant that appears in the Friedmann equation is [2]

$$G_C = \frac{1}{4\pi M_{\text{Pl}}^2(2 + 3\gamma + \beta)}, \quad (7)$$

while from the Newtonian limit, one can infer [23]

$$G_N = \frac{1}{8\pi M_{\text{Pl}}^2(1 - \alpha/2)}. \quad (8)$$

This effective gravitational constant affects the expansion rate during Big Bang Nucleosynthesis (BBN) with respect to the standard one. As a result the primordial helium abundance is modified by [24, 25]

$$\Delta Y_p = 0.08 \left(\frac{G_C}{G_N} - 1 \right). \quad (9)$$

Using the current bound $|\Delta Y_p| < 0.01$ (99.7% C.L.) [26–28] we obtain the following constraint

$$\left| \frac{\alpha + 3\gamma + \beta}{2 + 3\gamma + \beta} \right| < \frac{1}{8}. \quad (10)$$

4. Vacuum Cherenkov bounds. Photons could decay into spin-2 or spin-0 modes in vacuum when Lorentz symmetry is violated. Cosmic rays provide a lower bound on the speed of gravitational polarizations [29]. Specific constraints for Einstein-Æther theory have been derived in Ref. [30] and they exclude subluminal propagation to very high precision.² There is no detailed calculation for

Hořava gravity, or a quantitative translation of the Einstein-Æther results (given the similarity of the theories). However, the conservative expectation is that subluminal propagation is excluded to very high accuracy for both tensor and scalar polarisations.

5. ppN constraints. The two parametrized post-Newtonian (ppN) parameters which quantify preferred-frame effects are constrained by [31]

$$\begin{aligned} |\alpha_1| &< 10^{-4}, \\ |\alpha_2| &< 4 \times 10^{-7}. \end{aligned} \quad (11)$$

Using the weak field expressions for Hořava gravity [3, 32], these constraints translate into

$$\begin{aligned} \left| \frac{4(\alpha - 2\beta)}{1 - \beta} \right| &\lesssim 10^{-4}, \\ \left| \left(\frac{\alpha - 2\beta}{2 - \alpha} \right) \left(1 - \frac{(\alpha - 2\beta)(1 + \beta + 2\gamma)}{(1 - \beta)(\beta + \gamma)} \right) \right| &\lesssim 10^{-7}. \end{aligned} \quad (12)$$

6. Binary pulsars. The presence of a scalar polarisation can lead to dipolar emission and this would affect the dynamics of binary pulsars. The corresponding bounds are discussed in details in Ref. [33]. Figs. 1b and 8 of Ref. [33] present the constraints on the $\alpha = 2\beta$ plane. We will not reproduce these figures here. On this plane the ppN parameters α_1 and α_2 vanish but, as will be discussed below, considering this plane is no longer well motivated.
7. Black holes. The structure of isolated black holes can, in principle, provide constraints [34–37]. However, such constraints are significantly weaker than the binary pulsar constraints (at least on the $\alpha = 2\beta$ plane).
8. Gravitational Waves. The observation of the binary neutron star merger GW170817 with coincident gamma ray emission [6] yields

$$-3 \times 10^{-15} \leq c_T - 1 \leq 7 \times 10^{-16}, \quad (13)$$

which implies that

$$|\beta| \lesssim 10^{-15}. \quad (14)$$

Gravitational wave observations also provide a very mild lower bound on M_\star of the order of eV [7, 38]. This bound comes from considering the effect of the p^4 term in the dispersion relation (where p is momentum) that is suppressed by M_\star^2 .

² There is no compelling reason to *a priori* exclude superluminal propagation, unlike what seems to be suggested in Ref. [30]. Hence, vacuum Cherenkov constraints are one-sided.

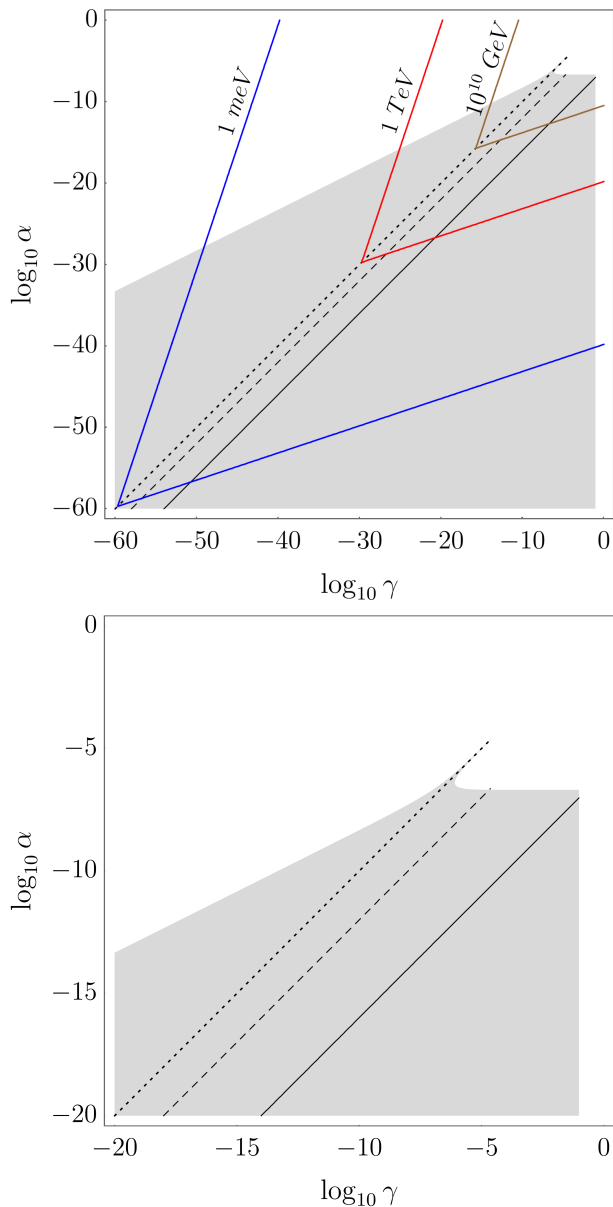


FIG. 1: The grey area shows the region of parameter space which is compatible with the constraints (Sec. (II)) for $\beta = 0$. The dotted, dashed and solid lines correspond to $c_S = 1$, $c_S = 10$ and $c_S = 1000$, respectively. The blue, red and brown lines show $M_{SC} = 1 \text{ meV}$, $M_{SC} = 1 \text{ TeV}$ and $M_{SC} = 10^{10} \text{ GeV}$. The lower panel focuses on the region $\alpha, \gamma > 10^{-20}$.

III. THEORETICAL CONSIDERATIONS AND INDIRECT CONSTRAINTS

The direct observational bounds obtained in the previous Section affect mainly the parameters of the IR effective theory. The parameters that become relevant in the UV are very weakly affected. In particular, gravitational waves provide only a very weak bound of M_\star , as mentioned above. In principle, laboratory test of gravity at

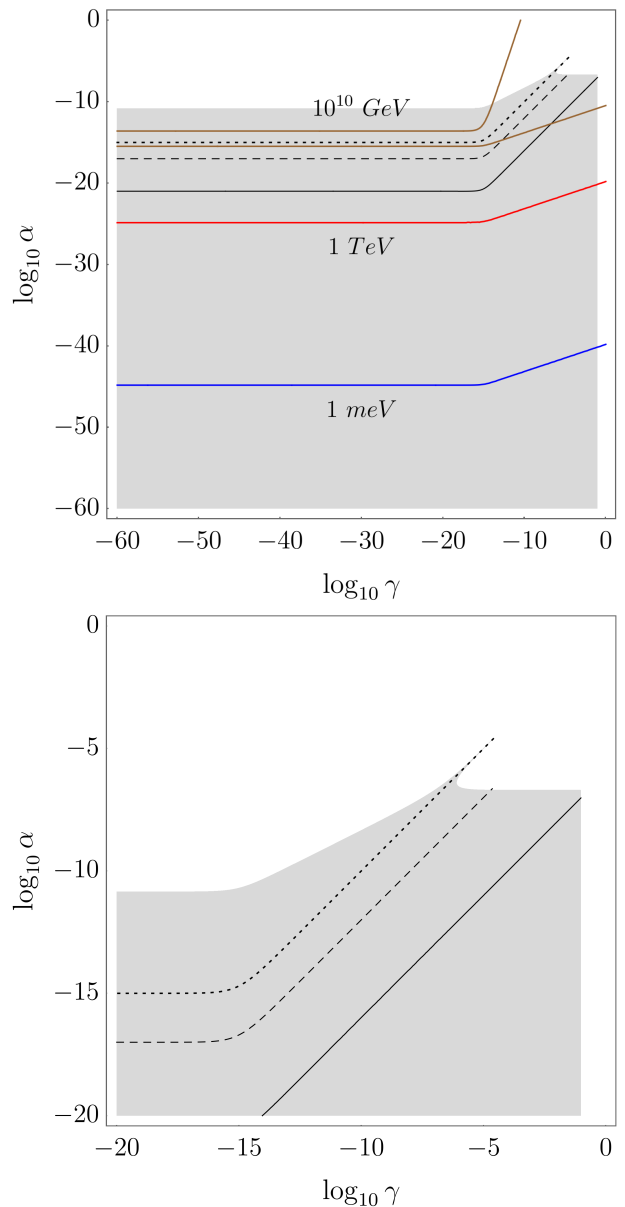


FIG. 2: Same as Fig. 1 but for $\beta = 10^{-15}$.

small length scales can place a direct lower bound on M_\star (as an energy scale), but current precision would place this bound in the meV range [39], so it would be also particularly weak.

Lorentz violations in the standard model are much more tightly constrained than in gravity. Indeed, if one were to assume that M_\star is a universal Lorentz violation scale for gravity and matter alike, then observations of the synchrotron radiation from the Crab nebula would require $M_\star > 2 \times 10^{16} \text{ GeV}$ [40]. However, if there is a mechanism to suppress the percolation of Lorentz symmetry breaking from gravity to matter, then there is no reason to believe that M_\star is a universal Lorentz violation scale. Moreover, such a mechanism seems to be necessary to keep Lorentz violation at bay already for lower mass

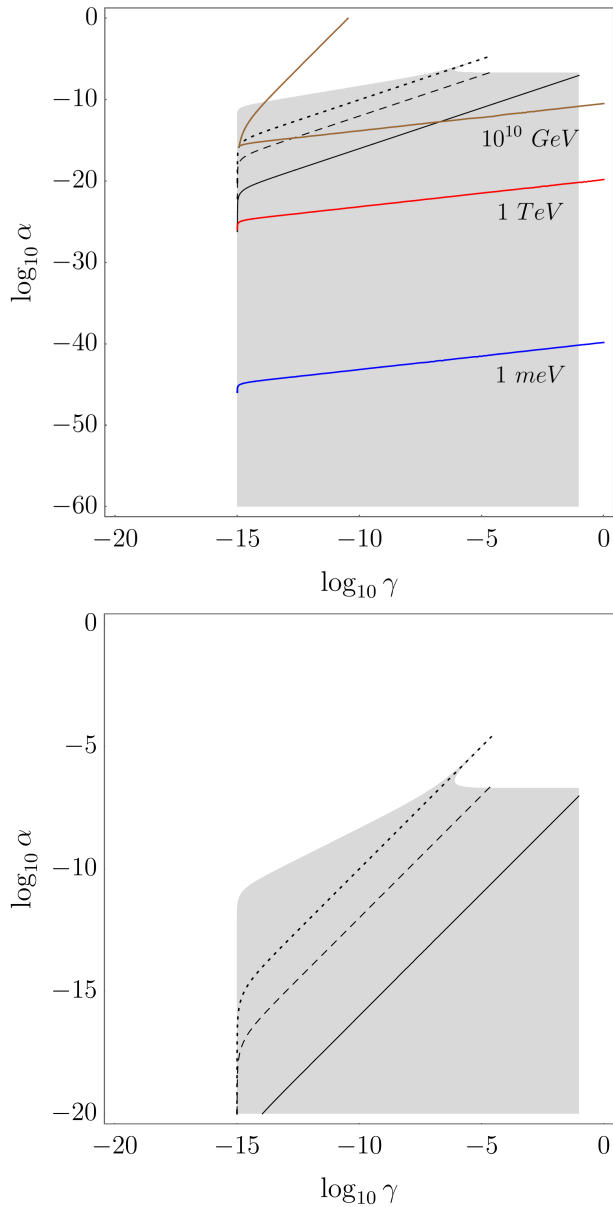


FIG. 3: Same as Fig. 1 but for $\beta = -10^{-15}$.

dimension operators in the standard model [41]. It has been suggested that the weak coupling between gravity and matter might suffice to suppress the percolation [42] but it is not clear how well this works in practice for Hořava gravity [43–45].

Irrespective of the details, it is intuitive that experiments will impose a lower bound on M_* . Interestingly, theoretical considerations can yield an upper bound. This is because the IR part of Hořava gravity, or equivalently action (2) exhibits strong coupling at a certain scale M_{SC} [20]. In particular, derivative self-interactions of the scalar mode compromise perturbativity. Provided that M_{SC} is sufficiently high, strong coupling is not an issue for infrared viability. However, power-counting renormalizability for Hořava gravity has been argued on the

basis of perturbativity and hence, strong coupling is a threat to the original motivation of the theory [20]. It turns out that having $M_* < M_{SC}$ — *i.e.* having the new physics coming from the UV completion kick in at low enough energies — can resolve the strong coupling problem [3]. However, as we will review in a bit more detail below, M_{SC} is controlled by the couplings of the infrared part of the action, (λ, ξ, η) or (α, β, γ) , which satisfy the bounds given in the previous section. Hence, M_* ends up having to satisfy an upper bound.

The M_{SC} can be calculated in the decoupling limit, which for action (2) corresponds to $M_{\mathcal{E}}^2 \rightarrow \infty$ while keeping $\alpha M_{\mathcal{E}}^2, \beta M_{\mathcal{E}}^2, \gamma M_{\mathcal{E}}^2$ fixed. This requires $\alpha, \beta, \gamma \rightarrow 0$. A detailed calculation was presented in Ref. [21]. It is worth pointing out that β has been set to zero there. This can indeed be done without loss of generality by a suitable time rescaling in action (1) or field redefinitions in action (2). We have avoided it here because the rescaling affects the various speeds. We discuss in more details how what we report corresponds to the results of Ref. [21] in Appendix B.

In the limit where $(\alpha, \beta, \gamma) \ll 1$ (consistent with the decoupling approximation), M_{SC} can have one of two different values depending on the magnitude of c_S^2 , namely

$$M_{SC} \approx \begin{cases} \sqrt{\alpha} M_{\mathcal{E}} c_S^{3/2} & , c_S^2 < 1 \\ \sqrt{\alpha} M_{\mathcal{E}} c_S^{-1/2} & , c_S^2 > 1 \end{cases} \quad (15)$$

It is worth stressing that when $(\alpha, \beta, \gamma) \ll 1$, and hence in the decoupling limit as well, $M_{\mathcal{E}} \sim M_p$ and both scales could be taken to be equal to the gravitational coupling scale as measured by experiments.

IV. ALLOWED REGION OF THE PARAMETER SPACE

Prior to GW170817, ppN constraints were considered particularly restrictive because, assuming that $\beta \ll 1$, they require $|\alpha - 2\beta| \lesssim 10^{-4}$. Hence, they restrict the 3-dimensional parameter space to a 2-dimensional surface with a width of 10^{-4} . All other constraints were either one-sided or weaker, so it was common practice to impose $\alpha = 2\beta$ in order to satisfy both ppN bounds to infinite accuracy (modulo tunings) and present graphically existing and new constraints on the $\alpha = 2\beta$ plane, see for example Refs. [33, 34, 37]. The double-sided constraint on c_T or β from GW170817 changes the picture drastically. The allowed region of the $\alpha = 2\beta$ plane shrinks to a line segment with width less than 10^{-15} , as one can always think of this plane as parametrised by β itself and some other combination of the couplings (*c.f.* with the figures in Refs. [33, 34, 37]).

Indeed it is now much more informative to focus on the $\beta = 0$ plane of the parameter space or, more generally, $|\beta| \sim 10^{-15}$ sections, as in Figs. 1-3. The top panel of each figure shows the whole parameter space and how some ad hoc choices for the strong coupling scale can restrict the parameter space. The lower panels zoom on

the region $\alpha, \gamma > 10^{-20}$. The dotted, dashed and solid lines correspond to choice of α and γ that lead to $c_S = 1$, $c_S = 10$ and $c_S = 1000$, respectively. They have been included to highlight that the c_S remains virtually unconstrained. It should be stressed that we have not imposed the vacuum Cherenkov constraints. Imposing them in a conservative fashion corresponds to excluding the part of the shaded region above the $c_S^2 = 1$ dotted line in each plot.

V. DISCUSSION

The detection of gravitational waves with an electromagnetic counterpart (GW170817), emitted by a binary neutron star merger, has put a stringent constraint on the speed of tensor modes. This translates to the tightest constraint so far in one of the parameters of Hořava gravity and it motivates revisiting the allowed region of the 3-dimensional parameter space. Instead of focussing on the 2-d plane that leads to exact agreement with general relativity in the weak field limit, as had been done in the literature so far, we focussed on 2-dimensional sections that satisfy the gravitational wave speed constraint and considered all other known constraints. The graphic representation of these constraints turns out to be quite illuminating in at least two respects:

1. The strong coupling scale M_{SC} is bounded from below by the Lorentz breaking scale M_\star , as discussed extensively in the text. M_\star is in turn bounded from below by observations that probe the higher order terms in the dispersion relation. Improving bounds on M_\star can hence reduce the parameter space significantly or rule out Hořava gravity entirely as a perturbatively renormalizable theory of gravity.
2. Even though the constraints on the parameters α , β and γ are very tight, the speed of the scalar polarisation remains virtually unconstrained. This stems from the type of dependence c_S has on these parameters and, more fundamentally, from the fact that the limit to general relativity is not smooth. Remarkably, even a very mild constraint on c_S would rule out a vast portion of the parameter space.

The aboves motivate finding novel ways to improve constraints on M_\star and c_S .

The prospects of measuring or constraining c_S and its importance in the context of Lorentz violations in gravity have been recently discussed in Ref. [7]. In the context of Hořava gravity, one could hope to obtain stricter bounds on M_\star and c_S by precise gravitational waveform modelling. In principle, trustworthy bounds on M_\star could also be obtained from nongravitational experiments (along the lines of Ref. [40]), if percolations of Lorentz symmetry from gravity to matter were well understood. Both prospects are quite challenging but would significantly enhance our understanding of the role of Lorentz symmetry in gravitation.

ACKNOWLEDGMENTS

We are grateful to Ted Jacobson and Tony Padilla for illuminating discussions. AEG acknowledges financial support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement 646702 "CosTes-Grav"). M.S. is supported by the Royal Commission for the Exhibition of 1851. TPS acknowledges funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement n. 306425 "Challenging General Relativity" and partial support from the STFC Consolidated Grant ST/P000703/1. TPS would also like to acknowledge networking support by the COST Action GWverse CA16104.

Appendix A: Correspondence with hypersurface orthogonal Einstein-Æther theory

The action for Einstein-Æther theory [4] is given by

$$S = \frac{M_{\text{Pl}}^2}{2} \int \sqrt{-g} d^4x [{}^{(4)}R - c_1 \nabla_\mu u_\nu \nabla^\mu u^\nu - c_2 (\nabla_\mu u^\mu)^2 - c_3 \nabla_\mu u_\nu \nabla^\nu u^\mu + c_4 u_\alpha u_\beta \nabla^\alpha u_\mu \nabla^\beta u^\mu], \quad (\text{A1})$$

where the norm of the æther field is constrained by $u^\mu u_\mu = -1$. If the æther field is hypersurface orthogonal it can simply be written as the (normalized) gradient of a single scalar:

$$u_\mu = - \frac{\nabla_\mu \phi}{\sqrt{-\nabla_\nu \phi \nabla^\nu \phi}}. \quad (\text{A2})$$

When this form of the vector field is imposed at the level of the action, *i.e.* before the variation, the c_1 term is no longer independent, and can be written as a combination of the c_3 and c_4 . The resulting action matches the one in Eq. (2), and the correspondence of parameters is

$$\alpha = c_1 + c_4, \quad \beta = c_1 + c_3, \quad \gamma = c_2. \quad (\text{A3})$$

Appendix B: Time rescaling and the value of β parameter

In action (1) one can set $\xi = 1$ by performing the time rescaling

$$d\tilde{t} = \sqrt{\xi} dt. \quad (\text{B1})$$

Given the correspondence of couplings in eq. (3), in the covariant picture of action (2) this maps to $\beta = 0$. Note that $\xi > 0$ (or $\beta < 1$) is required for stability, see Sec.

II. Time rescalings do not leave speeds invariant clearly and the specific one corresponds to choosing the time coordinate such that the speed of tensor gravitational waves is set to 1. This becomes clearer if one tries to set $\beta = 0$ directly in the covariant setup of action (2), as it requires a particular metric redefinition and a u_μ rescaling of the type discussed in Ref. [46]. Such redefinitions leave the action formally invariant and for the specific ones that leads to $\beta = 0$ the new metric defines the null propagation cones of spin-2 gravitons. It should be emphasized that the speed of light also changes and, assuming it was 1 initially, it becomes $\sqrt{1 - \beta}$ after either of the two equivalent procedures discussed above. The complete mapping of couplings is

$$\begin{aligned}\tilde{\alpha} &= \alpha, & \tilde{\gamma} &= \frac{\gamma + \beta}{1 - \beta} \\ \tilde{\beta} &= 0, & \tilde{M}_{\tilde{E}}^2 &= \sqrt{1 - \beta} M_{AE}^2,\end{aligned}\quad (\text{B2})$$

where an overtild denotes the new couplings.

Ref.[21] performs calculations in the decoupling limit but it resorts to this limit after having set $\tilde{\beta}$ to zero as described above. Hence, the limit is taken to be $\tilde{M}_{\tilde{E}}^2 \rightarrow \infty$ while keeping $\tilde{\alpha}\tilde{M}_{\tilde{E}}^2, \tilde{\gamma}\tilde{M}_{\tilde{E}}^2$ fixed. This implies that $\tilde{\alpha}, \tilde{\gamma} \rightarrow 0$ while there is no further explicit reference to β . The speed of the scalar polarisation in this limit is $\tilde{c}_{\text{dec}}^2 = \tilde{\gamma}/\tilde{\alpha}$. In the process of determining the smallest suppression scale for cubic interactions, and therefore identifying the strong coupling scale, the authors of Ref.[21] impose $\tilde{c}_{\text{dec}} \leq 1$, with the justification

that it is preferable to avoid superluminality. Under this assumption the strong coupling scale turns out to be

$$\tilde{M}_{SC} = \sqrt{\tilde{\alpha}} \tilde{M}_{\tilde{E}} \tilde{c}_{\text{dec}}^{3/2}, \quad (\text{B3})$$

as this scale has the largest positive exponent for \tilde{c}_{dec} . A minor points of caution is that, after the time rescaling (B1) the speed of light is no longer unity but $\sqrt{1 - \beta}$. Hence, superluminal propagation in the decoupling limit corresponds to $\tilde{c}_{\text{dec}}^2 > 1 - \beta$. More importantly, in a Lorentz violating theory with a preferred foliation there is no pertinent reason to exclude superluminal propagation. On the contrary, the vacuum Cherenkov bounds disfavour subluminal modes.

The decoupling limit as defined in Sec. III is $M_{\tilde{E}}^2 \rightarrow \infty$ while keeping $\alpha M_{\tilde{E}}^2, \beta M_{\tilde{E}}^2, \gamma M_{\tilde{E}}^2$ fixed. This requires $\alpha, \beta, \gamma \rightarrow 0$, so it does not correspond exactly to the limit taken in Ref.[21]. However, when $\beta \rightarrow 0$ there is perfect agreement. Moreover, $\tilde{c}_{\text{dec}}^2 \sim c_S^2$ and the first point of caution above becomes moot. Indeed, following the line of Ref. [21] but relaxing the assumption $\tilde{c}_{\text{dec}} \leq 1$, the strong coupling scale is

$$M_{SC} = \begin{cases} \sqrt{\alpha} M_{\tilde{E}} c_S^{3/2} (1 - \beta)^{3/4}, & c_S^2 (1 - \beta) < 1 \\ \sqrt{\alpha} M_{\tilde{E}} c_S^{-1/2} (1 - \beta)^{-1/4}, & c_S^2 (1 - \beta) > 1 \end{cases}$$

where we have purposefully not taken the limit $\beta \rightarrow 0$. These expressions agree with that of eq. (15) when $\alpha, \beta, \gamma \ll 1$.

-
- [1] P. Horava, Phys. Rev. D **79**, 084008 (2009) doi:10.1103/PhysRevD.79.084008 [arXiv:0901.3775 [hep-th]].
 - [2] D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. **104**, 181302 (2010) doi:10.1103/PhysRevLett.104.181302 [arXiv:0909.3525 [hep-th]].
 - [3] D. Blas, O. Pujolas and S. Sibiryakov, JHEP **1104**, 018 (2011) doi:10.1007/JHEP04(2011)018 [arXiv:1007.3503 [hep-th]].
 - [4] T. Jacobson and D. Mattingly, Phys. Rev. D **64**, 024028 (2001) doi:10.1103/PhysRevD.64.024028 [gr-qc/0007031].
 - [5] T. Jacobson, Phys. Rev. D **81**, 101502 (2010) Erratum: [Phys. Rev. D **82**, 129901 (2010)] doi:10.1103/PhysRevD.82.129901, 10.1103/PhysRevD.81.101502 [arXiv:1001.4823 [hep-th]].
 - [6] B. P. Abbott *et al.* [LIGO Scientific and Virgo and Fermi-GBM and INTEGRAL Collaborations], Astrophys. J. **848**, no. 2, L13 (2017) doi:10.3847/2041-8213/aa920c [arXiv:1710.05834 [astro-ph.HE]].
 - [7] T. P. Sotiriou, arXiv:1709.00940 [gr-qc].
 - [8] T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. Lett. **102**, 251601 (2009) doi:10.1103/PhysRevLett.102.251601 [arXiv:0904.4464 [hep-th]].
 - [9] S. Weinfurtner, T. P. Sotiriou and M. Visser, J. Phys. Conf. Ser. **222**, 012054 (2010) doi:10.1088/1742-6596/222/1/012054 [arXiv:1002.0308 [gr-qc]].
 - [10] P. Horava and C. M. Melby-Thompson, Phys. Rev. D **82**, 064027 (2010) doi:10.1103/PhysRevD.82.064027 [arXiv:1007.2410 [hep-th]].
 - [11] D. Vernieri and T. P. Sotiriou, Phys. Rev. D **85**, 064003 (2012) doi:10.1103/PhysRevD.85.069901, 10.1103/PhysRevD.85.064003 [arXiv:1112.3385 [hep-th]].
 - [12] D. Vernieri and T. P. Sotiriou, J. Phys. Conf. Ser. **453**, 012022 (2013) doi:10.1088/1742-6596/453/1/012022 [arXiv:1212.4402 [hep-th]].
 - [13] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S. M. Sibiryakov and C. F. Steinwachs, Phys. Rev. D **93**, no. 6, 064022 (2016) doi:10.1103/PhysRevD.93.064022 [arXiv:1512.02250 [hep-th]].
 - [14] T. P. Sotiriou, M. Visser and S. Weinfurtner, Phys. Rev. D **83**, 124021 (2011) doi:10.1103/PhysRevD.83.124021 [arXiv:1103.3013 [hep-th]].
 - [15] A. O. Barvinsky, D. Blas, M. Herrero-Valea, S. M. Sibiryakov and C. F. Steinwachs, arXiv:1706.06809 [hep-th].
 - [16] T. P. Sotiriou, M. Visser and S. Weinfurtner, JHEP **0910**, 033 (2009) doi:10.1088/1126-6708/2009/10/033 [arXiv:0905.2798 [hep-th]].
 - [17] C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, JHEP **0908**, 070 (2009) doi:10.1088/1126-6708/2009/08/070 [arXiv:0905.2579 [hep-th]].
 - [18] D. Blas, O. Pujolas and S. Sibiryakov, JHEP

- 0910**, 029 (2009) doi:10.1088/1126-6708/2009/10/029 [arXiv:0906.3046 [hep-th]].
- [19] K. Koyama and F. Arroja, *JHEP* **1003**, 061 (2010) doi:10.1007/JHEP03(2010)061 [arXiv:0910.1998 [hep-th]].
- [20] A. Papazoglou and T. P. Sotiriou, *Phys. Lett. B* **685**, 197 (2010) doi:10.1016/j.physletb.2010.01.054 [arXiv:0911.1299 [hep-th]].
- [21] I. Kimpton and A. Padilla, *JHEP* **1007**, 014 (2010) doi:10.1007/JHEP07(2010)014 [arXiv:1003.5666 [hep-th]].
- [22] S. Mukohyama, *Class. Quant. Grav.* **27**, 223101 (2010) doi:10.1088/0264-9381/27/22/223101 [arXiv:1007.5199 [hep-th]].
- [23] T. Jacobson, *PoS QG -PH*, 020 (2007) [arXiv:0801.1547 [gr-qc]].
- [24] X. I. Chen, R. J. Scherrer and G. Steigman, *Phys. Rev. D* **63**, 123504 (2001) doi:10.1103/PhysRevD.63.123504 [astro-ph/0011531].
- [25] S. M. Carroll and E. A. Lim, *Phys. Rev. D* **70**, 123525 (2004) doi:10.1103/PhysRevD.70.123525 [hep-th/0407149].
- [26] Y. I. Izotov, T. X. Thuan and N. G. Guseva, *Mon. Not. Roy. Astron. Soc.* **445** (2014) no.1, 778 doi:10.1093/mnras/stu1771 [arXiv:1408.6953 [astro-ph.CO]].
- [27] E. Aver, K. A. Olive and E. D. Skillman, *JCAP* **1507**, no. 07, 011 (2015) doi:10.1088/1475-7516/2015/07/011 [arXiv:1503.08146 [astro-ph.CO]].
- [28] C. Patrignani *et al.* [Particle Data Group], *Chin. Phys. C* **40**, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001
- [29] G. D. Moore and A. E. Nelson, *JHEP* **0109**, 023 (2001) doi:10.1088/1126-6708/2001/09/023 [hep-ph/0106220].
- [30] J. W. Elliott, G. D. Moore and H. Stoica, *JHEP* **0508**, 066 (2005) doi:10.1088/1126-6708/2005/08/066 [hep-ph/0505211].
- [31] C. M. Will, *Living Rev. Rel.* **9**, 3 (2006) doi:10.12942/lrr-2006-3 [gr-qc/0510072].
- [32] D. Blas and H. Sanctuary, *Phys. Rev. D* **84**, 064004 (2011) doi:10.1103/PhysRevD.84.064004 [arXiv:1105.5149 [gr-qc]].
- [33] K. Yagi, D. Blas, E. Barausse and N. Yunes, *Phys. Rev. D* **89**, no. 8, 084067 (2014) Erratum: [*Phys. Rev. D* **90**, no. 6, 069902 (2014)] Erratum: [*Phys. Rev. D* **90**, no. 6, 069901 (2014)] doi:10.1103/PhysRevD.90.069902, 10.1103/PhysRevD.90.069901, 10.1103/PhysRevD.89.084067 [arXiv:1311.7144 [gr-qc]].
- [34] E. Barausse, T. Jacobson and T. P. Sotiriou, *Phys. Rev. D* **83**, 124043 (2011) doi:10.1103/PhysRevD.83.124043 [arXiv:1104.2889 [gr-qc]].
- [35] E. Barausse and T. P. Sotiriou, *Phys. Rev. Lett.* **109**, 181101 (2012) Erratum: [*Phys. Rev. Lett.* **110**, no. 3, 039902 (2013)] doi:10.1103/PhysRevLett.110.039902, 10.1103/PhysRevLett.109.181101 [arXiv:1207.6370 [gr-qc]].
- [36] E. Barausse and T. P. Sotiriou, *Phys. Rev. D* **87**, 087504 (2013) doi:10.1103/PhysRevD.87.087504 [arXiv:1212.1334 [gr-qc]].
- [37] E. Barausse and T. P. Sotiriou, *Class. Quant. Grav.* **30**, 244010 (2013) doi:10.1088/0264-9381/30/24/244010 [arXiv:1307.3359 [gr-qc]].
- [38] B. P. Abbott *et al.* [LIGO Scientific and VIRGO Collaborations], *Phys. Rev. Lett.* **118**, no. 22, 221101 (2017) doi:10.1103/PhysRevLett.118.221101 [arXiv:1706.01812 [gr-qc]].
- [39] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl and S. Schlamminger, *Prog. Part. Nucl. Phys.* **62**, 102 (2009). doi:10.1016/j.pnpnp.2008.08.002
- [40] S. Liberati, L. Maccione and T. P. Sotiriou, *Phys. Rev. Lett.* **109**, 151602 (2012) doi:10.1103/PhysRevLett.109.151602 [arXiv:1207.0670 [gr-qc]].
- [41] R. Iengo, J. G. Russo and M. Serone, *JHEP* **0911**, 020 (2009) doi:10.1088/1126-6708/2009/11/020 [arXiv:0906.3477 [hep-th]].
- [42] M. Pospelov and Y. Shang, *Phys. Rev. D* **85**, 105001 (2012) doi:10.1103/PhysRevD.85.105001 [arXiv:1010.5249 [hep-th]].
- [43] M. Colombo, A. E. Gumrukcuoglu and T. P. Sotiriou, *Phys. Rev. D* **91**, no. 4, 044021 (2015) doi:10.1103/PhysRevD.91.044021 [arXiv:1410.6360 [hep-th]].
- [44] M. Colombo, A. E. Gumrukcuoglu and T. P. Sotiriou, *Phys. Rev. D* **92**, no. 6, 064037 (2015) doi:10.1103/PhysRevD.92.064037 [arXiv:1503.07544 [hep-th]].
- [45] A. Coates, M. Colombo, A. E. Gumrukcuoglu and T. P. Sotiriou, *Phys. Rev. D* **94**, no. 8, 084014 (2016) doi:10.1103/PhysRevD.94.084014 [arXiv:1604.04215 [hep-th]].
- [46] B. Z. Foster, *Phys. Rev. D* **72**, 044017 (2005) doi:10.1103/PhysRevD.72.044017 [gr-qc/0502066].